

Rebuild The Mathematical Structure for some Physical Phenomenon

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Abstract: The research is involved with reforming and developing some equations, which helps to increase the correctness of the outputs, and it concentrates on the mathematical component of some phenomena or behaviors of light. The reader will gain from it in addition to how it supports the explanation with a more thorough explanation. Each law or equation is explained with proofs, compared to its original form, and other equations with practical outputs are added. These equations are then connected to the traditional outputs. Additionally, the nature of light was explored, and in order to accommodate tests and occurrences, an amendment to the known nature of light was introduced. The modification was supported by accurate physical calculations and mathematically sound concepts. Since the study have vital information that is not common knowledge, rules and equations are significant from a variety of fields and in a wide range of sciences. The Compton equation and its , in addition to the Bragg equation, were created.

Keywords: Bragg's equation; Compton's equation; Reflection of light; The behavior of light; The nature of light.

1. Introduction

The study examines how light behaves from various perspectives, including refraction, reflection, etc. This study consists of three parts. The first section begins with dealing with the double slit experiment, correcting a mathematical error that caused the results of this experiment to be inaccurate, and creating equations that express the experiment and what is related to it. As a benefit of improving the mathematical understanding of this experiment, new variables and products were added. This part focused on the transition from abstract to concrete conceptions that express physical numbers and the value of trigonometry in expressing phenomena. The second component included the X-ray diffraction Bragg equation, which likewise contained a physical error that slightly influenced the Bragg equation findings, was repaired as simply and uncomplicatedly as possible by adding certain mathematical inputs. In order to accurately explain all circumstances, the equation was then developed in a more detailed and general way. In order to make this experiment the first of its kind, this section discussed one of the conclusions based on this occurrence, which is to find the approximate pattern of wavy lines. the final component included Compton's equation. This equation has the proper mathematical foundation and complete proof, but if we proof it in a different way, it produces an equation that is different from the one that is known, and all of this has contributed to improve of understanding of this equation.

2. Preliminaries

The light wavelength (λ): The distance separating two of a light wave's peaks or troughs is known as the light wavelength.

Frequency (f): The quantity of light waves that are created during a certain time period.

Amplitude (A): The height of the light wave.

Optical path (r): The displacement that light travels in a vacuum at the time the light travels a displacement that is always smaller than that traveled in a vacuum and equal to a multiple of the wavelength.

Visual trajectory difference (Δr): The difference between two visual trajectories, which is what we want to compute in this experience.

Average coefficient (n): The ratio of the speed of light in the concerned medium to its speed in a vacuum, or the ratio of the displacement traveled by the light in the same medium to the displacement traveled by the light in a vacuum during the same period of time.

Transmittance coefficient (n^{-1}): amount multiplied by the mean factor to make it equal to one, or multiplied by the speed of light in the medium to make the velocity equal to the speed of light in a vacuum, or multiplied by the displacement to make the displacement equal to the displacement in a vacuum, and equal to the inverted scale.

Phase difference angle (ϕ): is the angle which expresses the difference in the optical path at a point based upon the difference in the optical path.

The instantaneous optical path divergence angle (ϕ_n): is the angle between the zero angle and the angle 360° , used to specify the optical path divergence angle ignoring the delay between the two optical paths.

$$\phi_n = (\phi - 2s\pi) \quad \forall, \quad s = \max k, \quad k \leq \frac{\phi}{2\pi}, \quad k \in \mathbb{Z}^+ \quad (1)$$

Destructive and constructive interference: Destructive or destructive interference is the interference of two waves of light that are also of equal amplitude but different in phase by 180 degrees from each other. As a result, they cancel from one another. constructive interference of two waves of light so that they are equal in amplitude and the instantaneous phase difference angle is zero, and this results in the formation of a new wave of light with a new amplitude that is twice as great as the original wave of light.

Fringe separator (Δy): The distance between a bright fringe and an opaque fringe.

Diffraction angle (θ): The angle of diffraction of light from its straight path.

3. Materials and Methods

3.1 Double Slit Experiment

Dr. Thomas Young conducted this experiment as one of the tests to support his theory of light waves, and he used the results to determine the wavelength of light. The light source is positioned, a two-slot wall is set across from it, and a third wall is positioned opposite this light that has deviated a respectable distance in order to observe the existence of light and dark spots on the fourth wall. This phenomenon is dependent on the pattern of wave interference, which strengthened the theory of light waves, and waves' characteristics follow this. Due to the fact that this pattern is limited to the characteristics of the wave, which strengthened the hypothesis of

light waves, after that, the experiment was conducted on electrons, which caused the electron to exhibit a dual behavior. My study in this section will deal with a mathematical inaccuracy in experience, but there is still some uncertainty, and this situation is unrelated to my research.[1]

In order to do this experiment, a beam must be fired from one wall toward another wall that has two slits. From there, two rays must be released from the slits, but they must be diffracted in order to reach a third wall. A straight line is formed between the two pathways to measure the optical path difference, and this line is designated by the symbol d . This produces the first triangle, which is composed of the optical path difference, the straight line d , and the distance between the slits b . This triangle is isosceles, which means that it is equal in two angles and also contains lined, keeping in mind that the difference triangle in the optical route is a right triangle, and that the other triangle comprises the first path and a portion of the second ray, and they are equal, and if you know the angle of diffraction that corresponds to the difference in the optical path, such as the separation between the two slots, it is also possible to determine that difference. Then, you can determine the point range by approximating the sine of the angle of diffraction and its equivalent tangent to be equal to both. [2]

$$\Delta r = b \sin(\tan^{-1} L \setminus S) \quad , \quad L = S \tan(\tan^{-1} L \setminus S) \quad (2,3)$$

$$\Delta r = m\lambda = b \sin(\tan^{-1} L \setminus S) \quad (4)$$

Let $\Delta r = m\lambda$ for the Destructive interference This make $\Delta r = (m + 0.5)\lambda$ for the Destructive
Constructive

$$\sin(\tan^{-1} L \setminus S) \approx \tan(\tan^{-1} L \setminus S) \rightarrow \Delta L = \frac{S\lambda}{b} \quad (5)$$

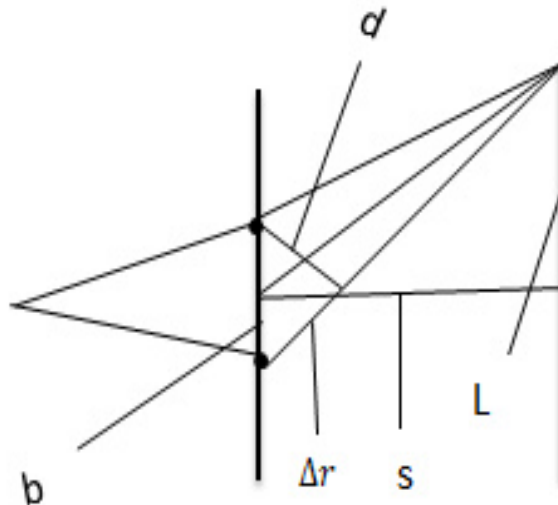


Figure 1: Double slit experiment.

This experience has two errors, one of which may be fixed by adding a condition and the other of which is a categorical error that cannot be ignored. The first mistake may be fixed by assuming that the diffraction angle and the angle between the slits and the straight line s are identical. [4] is feasible because the first error contains guides, and the guides assume that the two angles are equal. We must thus equalize them. [3] However, these two angles are difficult to measure, so I utilized a mathematical tool to create a condition with requirements that are both easy to measure and measurable. The second mistake is to assume that the triangle that contains the difference in the visible path is a right-angled triangle, which is incorrect because if it is a right-angled triangle, then the triangle next to it that contains the line s is also right-angled and the last one is isosceles and non-right angled. I'll fix this in a simple and clear way by introducing some variables, such as the source dimension. [5]

Δr : Optical path difference

θ_7 : diffraction angle

r : shortest optical path.

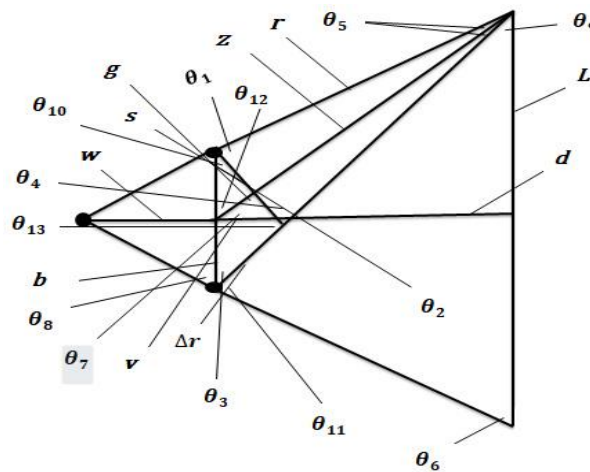


Figure 2: Mathematical structure of the double slit experiment.

In order to find the condition first, I will list some results based on the above drawing, which we can verify directly, and they are generally simple and intuitive, and the information is:

$$\theta_1 = \theta_4 = 90 - a, \quad \theta_5 = a, \quad \theta_7 = \tan^{-1} \frac{L}{d}, \quad \theta_{10} = 90 + a - \tan^{-1} \frac{2w}{b} = \tan^{-1} \frac{L}{d}, \quad \tan \theta_7 = \frac{L}{d} = \frac{s}{2g}$$

$$s = 2rsina, \quad 2g = b \sin \theta_7, \quad \theta_{13} = 90 + a, \quad \theta_2 = \theta_8 = \tan^{-1} \frac{2w}{b}, \quad \theta_8 + \theta_3 + \theta_{11} = 180, \quad \theta_6 = \theta_8, \quad \theta_{10} + \theta_{13} + \theta_3 = 180, \quad \theta_{10} + \theta_1 + \theta_2 = 180 \rightarrow \theta_{10} = 90 + a - \tan^{-1} \left(\frac{2w}{b} \right) \rightarrow a = \tan^{-1} \left(\frac{L}{d} \right) + \tan^{-1} \left(\frac{2w}{b} \right) - 90$$

$$\theta_{11} + \theta_6 + \theta_9 = 180, \quad \theta_7 = \theta_{10}, \quad \theta_7 = 90 - \theta_{12}, \quad \theta_9 = \tan^{-1} \left(\frac{2w}{b} \right) - 2a$$

$$\text{When } \theta_{10} + \theta_{13} + \theta_3 = 180 \rightarrow \theta_3 = \tan^{-1} \left(\frac{2w}{b} \right) - 2a$$

$$\text{Or } \theta_8 + \theta_3 + \theta_{11} = 180 \rightarrow \theta_3 - \tan^{-1} \left(\frac{2w}{b} \right) + 2a - \tan^{-1} \left(\frac{2w}{b} \right) + \tan^{-1} \left(\frac{2w}{b} \right) = 0$$

$$\rightarrow \theta_3 = \tan^{-1} \left(\frac{2w}{b} \right) - 2a$$

Considering that I'll assume the reader has a solid foundation in mathematics and that the narrative can generally make any unintelligible step clear.

The condition will be stated upfront.

We can extract the condition by taking use of what was just write.

$$\frac{L}{d} = \frac{2g}{s} \rightarrow \frac{L}{d} = \frac{b \sin \theta_7}{2r \sin a} \rightarrow \frac{L}{d} = \frac{b \sin \left(\tan^{-1} \frac{L}{d} \right)}{-2r \cos \left(\tan^{-1} \left(\frac{L}{d} \right) + \tan^{-1} \left(\frac{2w}{b} \right) \right)}$$

$$\frac{L}{d} = \frac{b \sin \left(\tan^{-1} \frac{L}{d} \right)}{\left(\sqrt{4w^2 + b^2} - 2(d+w) \csc \left(\tan^{-1} \frac{2w}{b} \right) \right) \cos \left(\tan^{-1} \left(\frac{L}{d} \right) + \tan^{-1} \left(\frac{2w}{b} \right) \right)}$$

$$1 = \frac{db \sin \left(\tan^{-1} \frac{L}{d} \right)}{L \left(\sqrt{4w^2 + b^2} - 2(d+w) \csc \left(\tan^{-1} \frac{2w}{b} \right) \right) \cos \left(\tan^{-1} \left(\frac{L}{d} \right) + \tan^{-1} \left(\frac{2w}{b} \right) \right)} \quad (6)$$

Consequently, this is the condition that permits the selection of the diffraction angle ($\tan^{-1} \frac{L}{d}$). is the same as the angle of the triangle in question (θ_{10}). I made an effort to express the condition as simply as feasible in terms of length and the convenience of measuring the variables it includes.

The second error, which is to assume the triangle in question is right-angled when it is not, is an absolute mistake, as I have described. It was necessary to establish a law for optical path divergence, which was predicated on the triangle's right angle. The source dimension, which was employed in the new formulation, is the most significant of the new additions to the variables. A valid formulation that regards the triangle as non-right-angled has been established with the aid of trigonometry and its rules. The other laws will rely on the same idea and the new optical path difference law, and I will also depend on trigonometry's principles.

The explanation leads us to the conclusion:

Knowing the sine rule in triangles is important in order to determine the difference in the optical path.

$$\frac{\Delta r}{\sin(\theta_{10})} = \frac{b}{\sin(\theta_{13})} \rightarrow \Delta r = \frac{b \sin \left(\tan^{-1} \frac{L}{d} \right)}{\sin \left(\tan^{-1} \left(\frac{L}{d} \right) + \tan^{-1} \left(\frac{2w}{b} \right) \right)} \quad (7)$$

We can distinguish the most important difference between the new formulation and the previous one. $\Delta r = b \sin \left(\tan^{-1} \frac{L}{d} \right)$.

Now I will find (a) by:

$$\sin \theta_9 = \sin \left(\tan^{-1} \left(\frac{2w}{b} \right) - 2a \right) = \frac{r \sin \left(\tan^{-1} \frac{2w}{b} \right)}{r + \Delta r}$$

$$a = \frac{\tan^{-1}\left(\frac{2w}{b}\right) - \sin^{-1}\left(\frac{r \sin\left(\tan^{-1}\frac{2w}{b}\right)}{r + \Delta r}\right)}{2}$$

$$a = \frac{\tan^{-1}\left(\frac{2w}{b}\right) - \sin^{-1}\left(\frac{r \sin\left(\tan^{-1}\frac{2w}{b}\right)}{r + \Delta r}\right)}{2} \quad (8)$$

3.2. Bragg equation for x-ray diffraction

This law relates to the diffraction of X-rays, and it can basically be proven in a very simple way, but it has some physical and mathematical errors, which are seen as I explained in the previous part. The visual path is as follows: [6]

$$\text{Previous equation } n\lambda = 2d\sin\theta$$

An illuminated spot occurs on a surface when n is a positive integer constructive interference.

A luminous dot does not emerge on a surface when n is a positive integer and half . [7]

As for the errors, they are as follows:

- 1 . The law does not take into account the refraction of the wave, even if its refraction is small, it is taken into account, and this will benefit us in the possibility of applying this new law in the phenomenon of interference in thin films, or even in the single-slit experiment (diffraction).
- 2 . The law is based on assumptions that are not mathematically proven and therefore is incorrect.

The following diagram shows the mathematical structure of the law.

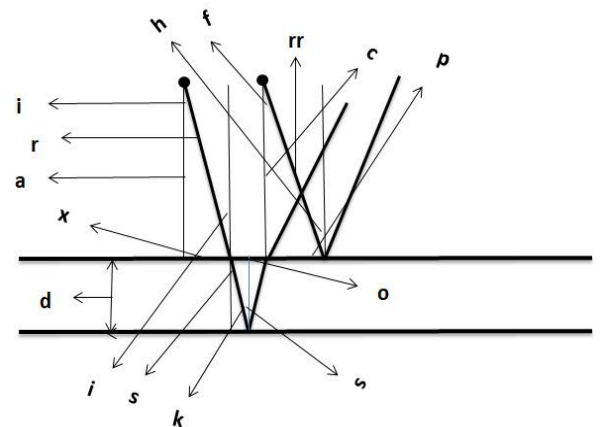


Figure 3: The mathematical structure of the experiment.

Note: The reflection beam is always represented by the letters p and c.

Through the above figure and our knowledge of some laws, we conclude the following:

$$\lambda_n = \frac{\lambda}{n}, \quad r = \frac{a}{\sin \tan^{-1}\left(\frac{a}{x}\right)}, \quad i = \tan^{-1}\left(\frac{x}{a}\right), \quad f = \tan^{-1}\left(\frac{p}{c}\right), \quad rr = \frac{c}{\sin \tan^{-1}\left(\frac{c}{p}\right)}$$

We find point 1 through the rules of triangles and taking into account refraction, as will be shown:

$$\frac{n_2}{n_1} = n = \frac{\sin\left(\tan^{-1}\frac{a}{x}\right)}{\cos s} \rightarrow 2k = \frac{2dn}{\sin\left(\tan^{-1}\frac{a}{x}\right)} \rightarrow 2k = \frac{2dn^2}{\sin\left(\tan^{-1}\frac{a}{x}\right)}$$

Note: refraction has been taken into account. As for point 2, we find it as follows.

We find the time that the first path takes inside the crystal and multiply it by the speed of light.

$$out = \frac{2dn^3}{\sin\left(\tan^{-1}\frac{a}{x}\right)}$$

$$\Delta r = \frac{2dn^2}{\sin\left(\tan^{-1}\frac{a}{x}\right)}(n - 1) \quad (9)$$

Note: the rays come from the air, so the refractive index is the same

3.3. Compton's equation

The Nobel Prize for science was given to the physicist Arthur Holly Compton in 1923 for his discovery of the Compton effect. The Compton equation looks like this: [8]

$$\lambda' - \lambda = \frac{h}{cm_e}(1 - \cos \theta) \quad (10)$$

But how does the electron gain energy? And how much does he earn? Upon collision, it is assumed that the electron takes energy from the photon, but to what extent does it take from it? an amount sufficient to escape from the crystal, or does it take from it to the extent that the photon has enough energy to escape. To answer this important question, we will use some ideas without study the difference in how close the electron is from the surface. [9]

The first assumption: since the kinetic energy of the electron increases (presumably) with the increase in the energy of the photon, this means that any scattered photon has the same amount of energy, no matter how different the frequency of the original incident light is, and both the photon and the electron cannot share energy because there is no clear physical reason for that, in addition to There is nothing that requires that, so if we assume that they share, and supposing a photon of energy 5 falls, this means that it will come out with 2.5, and on the same metal a photon of energy 6 fell on it, it will come out with 3, so in both cases it came out, so there is nothing that depends on half of the energy. The emitted photon has an amount of energy close to the energy of the resting electron. Below is the proof:

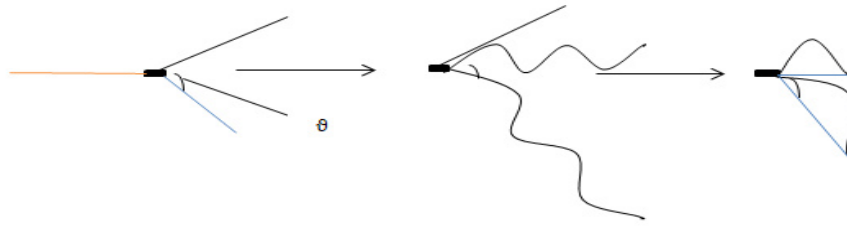


Figure 5: Light scattering and wavelength change.

The result is as follows.

$$\lambda' = \frac{h}{p'} \quad , \quad \lambda = \frac{h}{p} \cos \vartheta \quad \rightarrow \quad \lambda' - \lambda = \frac{h}{p} (1 - \cos \vartheta) \rightarrow p' = cm_e \rightarrow cp' = c^2 m_e$$

$$E_e = m_e c^2 = cp' = E_p \quad (11)$$

Note: The triangle above may not be completely straight, but the purpose of proof is to show the photon energy is close to the electron energy.

(The energy of the static electron is equal to the energy of the photon that the crystal emits, which has the original frequency and can expel electrons from the crystal).

Or, (The energy of the ejecting electron is the same as the energy of the ejecting photon, minus the energy that both lose in the process of ejecting from the crystal).

(No matter how diverse the light is, photon energy remains constant).

The second assumption: It is the exact opposite of the first, that is, the electron acquires only what is sufficient to escape, whatever the frequency of the light, and then the energy of the photon is variable.

to separate them and determine which of the two possibilities is correct through experiment or finding a mathematical or physical idea.

$$\frac{(hf - hf' + cp')^2 - (cp')^2}{c^2} = (p - p')^2$$

$$\rightarrow (hf)^2 - (hf')^2 = (hf - hf')^2 \rightarrow \lambda = \lambda' ?!!$$

And this is not a problem, but we have an error in the multiplication, so we must add the cos function for multiplying the momentum, but where is the momentum? Even if it do not appears, as long as there is an valu related to it, it must be added, so the equation will be like this:

$$\frac{\lambda}{\lambda'} = \cos \vartheta \quad \text{and from } \lambda' = \frac{h}{p'}$$

$$(hf)^2 - (hf')^2 = (hf - hf')^2 \rightarrow hf' = hf \cos \vartheta \rightarrow \lambda = \lambda' \cos \vartheta$$

That rule is spelled out as follows

$$\text{let } a \text{ and } a_1 \text{ when } a \cdot a_1 = a \cdot a_1 \sin \theta \quad \text{and } a = \frac{b}{c} \quad , \quad a_1 = \frac{b_1}{c_1}$$

$$\text{so } c \cdot c_1 = c \cdot c_1 \csc \theta \quad \text{because } c \cdot c_1 = \frac{bb_1}{a \cdot a_1 \sin \theta} = c \cdot c_1 \csc \theta$$

If b were not an unrelated quantity, we would treat it in accordance with the rule.

What is the light's mass? One of the most well-known questions that certainly calls for understanding the nature of light is this one. We arrived to the conclusion that light is a particle known as a photon that moves in the shape of a wave after formulating ideas on the nature of light. So, What is the mass of a photon and how can it be calculated?

$$\text{we know } p = \frac{h}{\lambda} \rightarrow m = \frac{h}{c\lambda}$$

Although I am aware that the photon mass is not constant, I shall still refer to it as mass.

The mass of the photon was easily calculated, but there is a criticism that the relationship between mass and speed is inverse. We are aware that this is a very logical argument, but it is evident when we state, for instance, that an item with a mass of 5 grams gains mass as it moves faster (by increasing the load on it or by building it) So its speed diminishes, but what causes the mass of light to increase as the velocity of light lowers? For instance, light's mass increases when its velocity falls as it travels through a clear material. But before the answer, we may state that the wavelength lengthens to make up for the speed's decline, The mass increases, whether the light wavelength decreasing or constant, just as the light wavelength cannot rise.

This increases the mass whether the wavelength decreases or stays constant and cannot be raised since it is exactly proportional to the speed, therefore it will decrease in accordance with the connection between them. Additionally, it is not feasible to modify the frequency to increase it. However, since we do not know whether the wavelength changes or stays the same, the mass equation will be created to account for mass from the new velocity and wavelength. To explain this occurrence to us, something more must be added to our understanding of the nature of light. Numerous phenomena related to light are caused by photons, which are little particles that move like waves. (Light is a stream of photons that travel in a single waveform and has the forces of wave aggregation and expansion.) The following equation illustrates the strong relationship between the photon and the electron. Since the speed of light is inversely related to the force of aggregation and directly related to the force of expansion, if the speed of light and the speed of the electron are equal, then the mass of photon will be equal to the mass of the electron.

$$\Delta m = \frac{h}{c_f \lambda_f} - \frac{h}{c_i \lambda_i} = \frac{hc_i \lambda_i - hc_f \lambda_f}{c_i \lambda_i c_f \lambda_f} = -\frac{h \Delta c \lambda}{c_i \lambda_i c_f \lambda_f} \quad (12)$$

Some useful terms

$$\text{Adjusted wavelength : } \lambda_r = \frac{c_i \lambda_i}{c_f} \quad , \quad \text{The corrected speed : } c_r = \frac{c_i \lambda_i}{\lambda_f}$$

This subject is over in terms of physics, but the mathematical side is still relevant, and that's what I'm most interested in. Everything is used as a concept before being completely executed. so The meaning being applied as a concept, and those phrases (mass, velocity...), do not necessarily match the images we construct in our mind, but they may be in any shape or anything, and we know that whatever that form is, it satisfies those requirements. Do not be astonished when I say that the concept applies to most ideas, and we don't know that, in particular, the explanations offered by quantum theory. All of his equations convey the concept, not always the shape or image, as well as the general theory of relativity, in which the physicist Einstein presupposed the

existence of the space-time tissue, which we treat as a flat surface. We're merely utilizing those formulae, and it may be completely different.

1. Results and Conclusions

A summary of the results of this work is as follows:

1. Rewriting the rules of the double slit experiment and include helpful new ideas.
2. Making Bragg's diffraction law accurate and formulate it in a way that takes into account all variables, and using it to determine the shape of waves on a surface.
3. Expanding Compton's Law, fill in any holes, and come to fresh conclusions.
4. describing the nature of light.

There have been experiments into certain light behaviors, and equations expressing these phenomena have been created. All equations were evaluated mathematically and physically, and I left them free of any flaws or problems so that they could be utilized to produce accurate and reliable results. I also finished fixing several equations and structuring them in the simplest and easiest method possible.

References

- [1] LF Granz, O. Mathia, S. Villalba-Chávez, C. Müller, " Electron-positron pair production in oscillating electric fields with double-pulse structure." *Physics Letters B*, vol.78, no.10, pp. 85-89, 2019.
- [2] Jau Tang, Z. B. Hu, " Analysis of single-photon self- interference in Young's double-slit experiments," *Results in Optics*, vol.9, no.1, pp. 100-102, 2022.
- [3] Itzhak Orion and M. Laitman, "The Double-Slit Experiment and Wave-Particle Duality: Toward a Novel Quantum Interpretation," *Journal of Modern Physics*, vol.1,no.1, pp. 90-92, 2010.
- [4] Mohamed Abdel-Basset, Doaa El-Shahat, Mohammed Jameel, Mohamed Abouhawwash," Young's double-slit experiment optimizer: A novel metaheuristic optimization algorithm for global and constraint optimization problems," *Computer Methods in Applied Mechanics*, vol.403,no.1, pp. 115-117, 2023.
- [5] S. Sorek . Moshe Wallace Callen," Explaining the Double-Slit Experiment" *Journal of Modern Physics*, vol.2 no.1, pp. 30-35, 2011.
- [6] G. E. M. Jauncey, "The Scattering of X- Rays and Bragg's Law," *PNAS*,vol.10, no.2, pp: 57-60, 2023.
- [7] Yuyao Nie, Lei Dong, Jie Wu, Dejun Li A," Investigation of Ti-6Al-4V/ZrB2 trilaminar structure and its high temperature stability," *Ceramics International*, vol.44, no.15, pp. 76-82, 2019.



[8] Ozgur Ozcan, Cem Gercek,” Students' Mental Models of Light to Explain the Compton Effect,” *Procedia - Social and Behavioral Sciences*, vol.191, no.2, pp. 2195-2197, 2015.

[9] Ziya Saglam, Gokhan Sahin, Bahadir Boyacioglu,” Compton effect in terms of spintronic,” *Results in Physics*, vol.6, no.1, pp. 726-727, 2016.